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M.Sc. Sem III 12th Paper

FLUID MECHANICS

velocity $\vec{q}$ is a function of $\vec{r}, t$.

$$\Rightarrow \vec{q} = q(\vec{r}, t) \quad \text{———} (1)$$

$$\Rightarrow \vec{q} + \delta\vec{q} = q(\vec{r} + \delta\vec{r}, t + \delta t) \quad \text{———} (2)$$

Subtracting (1) from (2), we have

$$\Rightarrow \delta\vec{q} = \vec{q}(\vec{r} + \delta\vec{r}, t + \delta t) - \vec{q}(\vec{r}, t)$$

$$= \left[\vec{q}(\vec{r} + \delta\vec{r}, t + \delta t) - \vec{q}(\vec{r}, t + \delta t)\right] + \left[\vec{q}(\vec{r}, t + \delta t) - \vec{q}(\vec{r}, t)\right]$$

$$\Rightarrow \delta\vec{q} = \delta\vec{r} \cdot \nabla\vec{q}(\vec{r}, t + \delta t) + \delta t \cdot \frac{\partial\vec{q}(\vec{r}, t)}{\partial t}$$

Dividing by δt

$$\Rightarrow \frac{\delta\vec{q}}{\delta t} = \frac{\delta\vec{r}}{\delta t} \cdot \nabla\vec{q}(\vec{r}, t + \delta t) + \frac{\partial\vec{q}(\vec{r}, t)}{\partial t}$$

Take $\lim_{\delta t \to 0}$ both sides

$$\Rightarrow \frac{d\vec{q}}{dt} = \frac{d\vec{r}}{dt} \cdot \nabla\vec{q} + \frac{\partial\vec{q}}{\partial t}$$

$$\Rightarrow \frac{d\vec{q}}{dt} = \vec{q} \cdot \nabla\vec{q} + \frac{\partial\vec{q}}{\partial t}$$

$$\Rightarrow \frac{d\vec{q}}{dt} = (\vec{q}\cdot\nabla)\vec{q} + \frac{\partial \vec{q}}{\partial t} \quad \text{---(1)}$$

This equation gives the acceleration at any point P.

Q. For the motion of a perfect fluid, obtain the components of acceleration in rectangular Cartesian coordinates.

Soln Let u, v, w be the components of velocity $\vec{q}$ in the different axes.

$$\Rightarrow \vec{q} = u\vec{i} + v\vec{j} + w\vec{k}$$

$$\because \nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

$$\Rightarrow \vec{q}\cdot\nabla = (u\vec{i} + v\vec{j} + w\vec{k})\cdot\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$$

$$= u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Also, $\frac{d\vec{q}}{dt} = \frac{d}{dt}(u\vec{i} + v\vec{j} + w\vec{k}) = \frac{du}{dt}\vec{i} + \frac{dv}{dt}\vec{j} + \frac{dw}{dt}\vec{k}$

and $(\vec{q}\cdot\nabla)\vec{q} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)(u\vec{i} + v\vec{j} + w\vec{k})$

$$\frac{\partial \vec{q}}{\partial t} = \frac{\partial}{\partial t}(u\vec{i} + v\vec{j} + w\vec{k}) = \frac{\partial u}{\partial t}\vec{i} + \frac{\partial v}{\partial t}\vec{j} + \frac{\partial w}{\partial t}\vec{k}$$

So, eq(1) becomes

$$\frac{du}{dt}\vec{i} + \frac{dv}{dt}\vec{j} + \frac{dw}{dt}\vec{k} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)(u\vec{i} + v\vec{j} + w\vec{k})$$
$$+ \left(\frac{\partial u}{\partial t}\vec{i} + \frac{\partial v}{\partial t}\vec{j} + \frac{\partial w}{\partial t}\vec{k}\right)$$

Equating the coefficients of $\vec{i}, \vec{j}, \vec{k}$ on both sides, we get

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z}$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial z}$$

$$\frac{d\omega}{dt} = \frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} + \omega\frac{\partial \omega}{\partial z}$$

The above equations are ~~the~~ required components of acceleration in rectangular cartesian coordinates.